Fair Transit Stop Placement: A Clustering Perspective and Beyond

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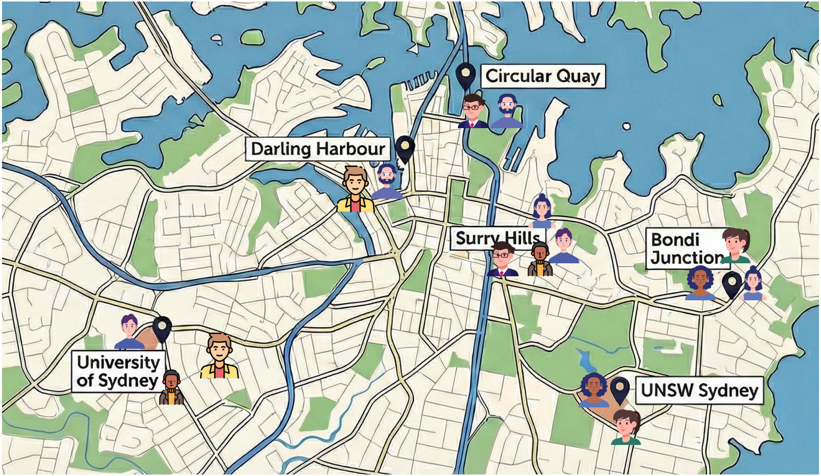
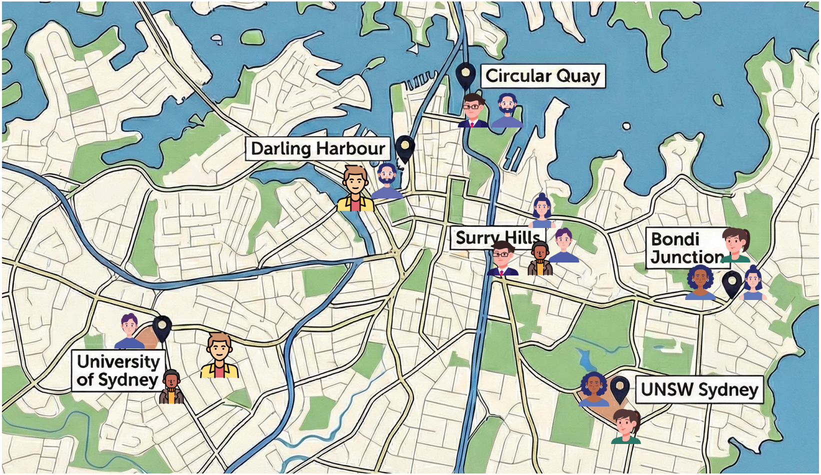
ICML 2026 (Seoul, South Korea)

Transit Stop Placement (TrSP)



Transit Stop Placement (TrSP)

: I travels from Darling Harbour to Uni.Syd.



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Select k-stops from the candidate stops for a shuttle bus .

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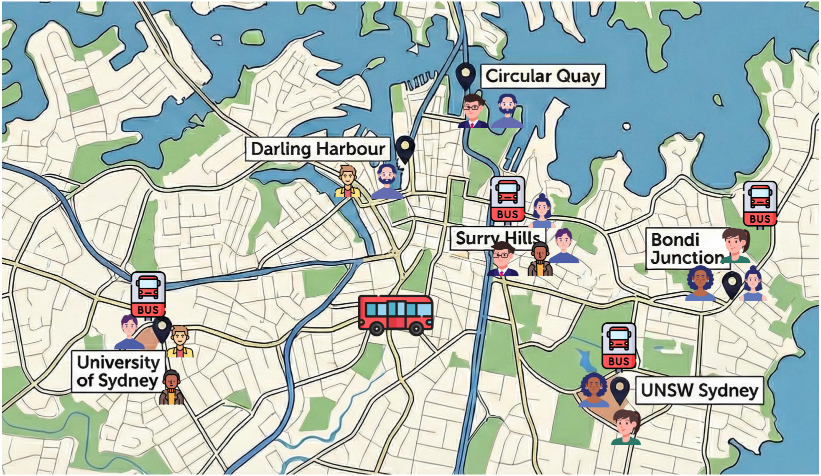
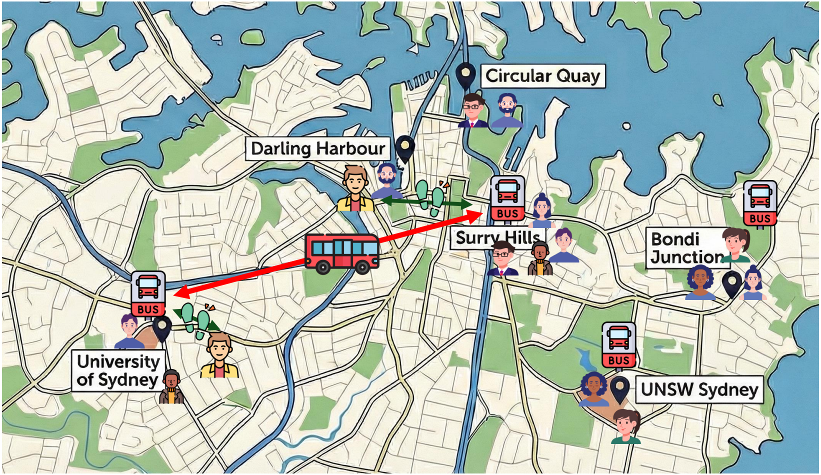
Example: a placement of 4 stops  .



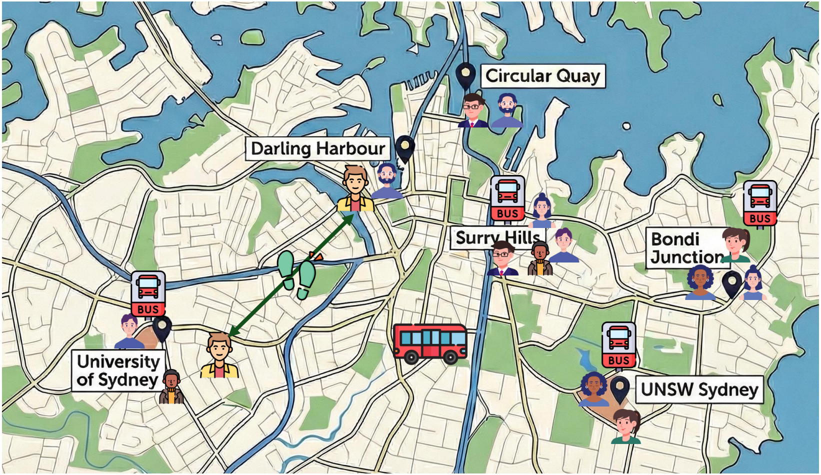
Transit Stop Placement (TrSP)

 : I can travel by the transit system.



Transit Stop Placement (TrSP)

 : I can also travel by foot.



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- A TrSP instance is given by $I = \langle N, \mathcal{C}, \Theta = \{\theta_i\}_{i \in N}, k \rangle$.
- A set of n agents $N := \{1, 2, \ldots, n\}$.
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Output: A (placement) solution $Y \subseteq \mathcal{C}$, $|Y| \leq k$.

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The **cost** of agent i with $\theta_i = (a_i, b_i)$ is given by

$$c_i(Y) = \min\Bigg\{ \underbrace{d(a_i, b_i)}_{\text{walking cost}},\ \underbrace{\min_{y_1, y_2 \in Y} \Big[d(a_i, y_1) + d'(y_1, y_2) + d(y_2, b_i) \Big]}_{\text{transit cost}} \Bigg\}.$$

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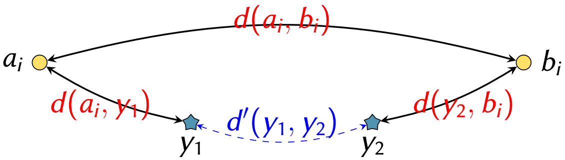

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The initial model was first studied in the *line* space [BEL25].

TrSP: Fairness Objectives

β-Justified Representation (β-JR)

Let $\beta \geq 1$. A solution $Y \subseteq \mathcal{C}$ is said to provide β-JR if for any subset of agents $S \subseteq N$ with $|S| \geq \frac{2n}{k}$, and **every pair of transit stops** $T \subseteq \mathcal{C}, |T| = 2$, there exists an agent $i \in S$ such that $\beta \cdot c_i(T) \geq c_i(Y)$.

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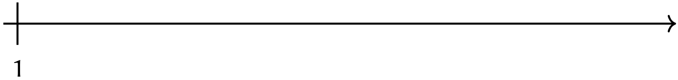
(α, β)-core

Let $\alpha, \beta \geq 1$. A solution $Y \subseteq \mathcal{C}$ is in the (α, β)-core if for every subset of agents $S \subseteq N$, and **every transit stop set** $T \subseteq \mathcal{C}$ with $|S| \geq \alpha \cdot |T| \cdot \frac{n}{k}$, there exists an agent $i \in S$ such that $\beta \cdot c_i(T) \geq c_i(Y)$.

Our Results: JR Approximation



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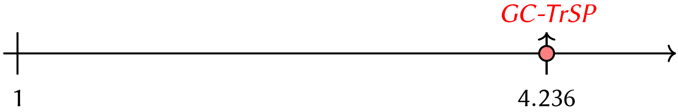

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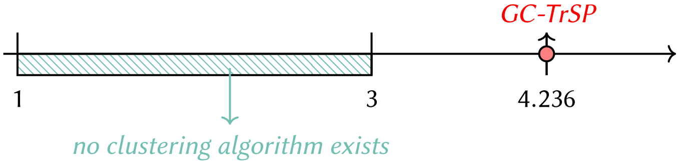

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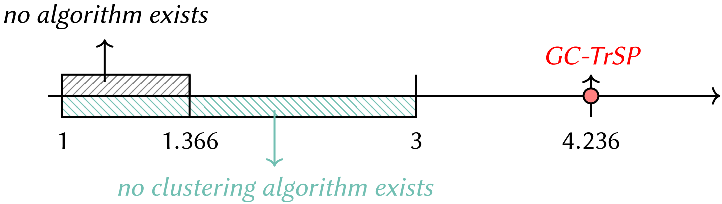

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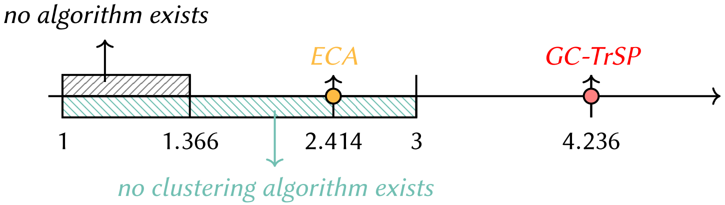

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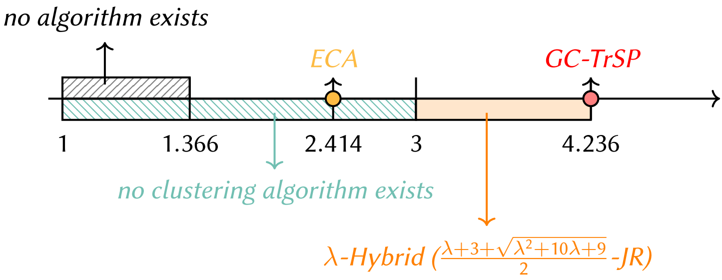

Our Results: Core Approximations

	core	
	Lower Bound	Upper Bound
GC-TrSP	$(2, 1+\sqrt{2})$	$(2, 1+\sqrt{2})$
ECA	(α, β)-core for any $\alpha, \beta \geq 1$	–
λ-**Hybrid**	$(2, \frac{\sqrt{4\lambda^2+12\lambda+1}+2\lambda+1}{4\lambda})$	$(2, \frac{\sqrt{\lambda^2+6\lambda+1}+\lambda+1}{2\lambda})$

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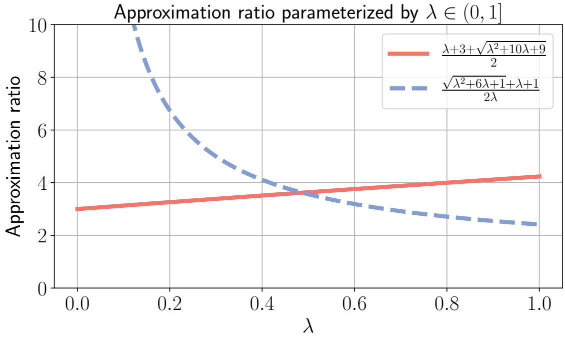

Poster Session Information

Poster: **Thu, 9th July 5:00PM – 6:45PM** at **COEX: HALL A**

Paper link: https://arxiv.org/abs/2602.06776

Reference

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