Fair Transit Stop Placement: A Clustering Perspective and Beyond

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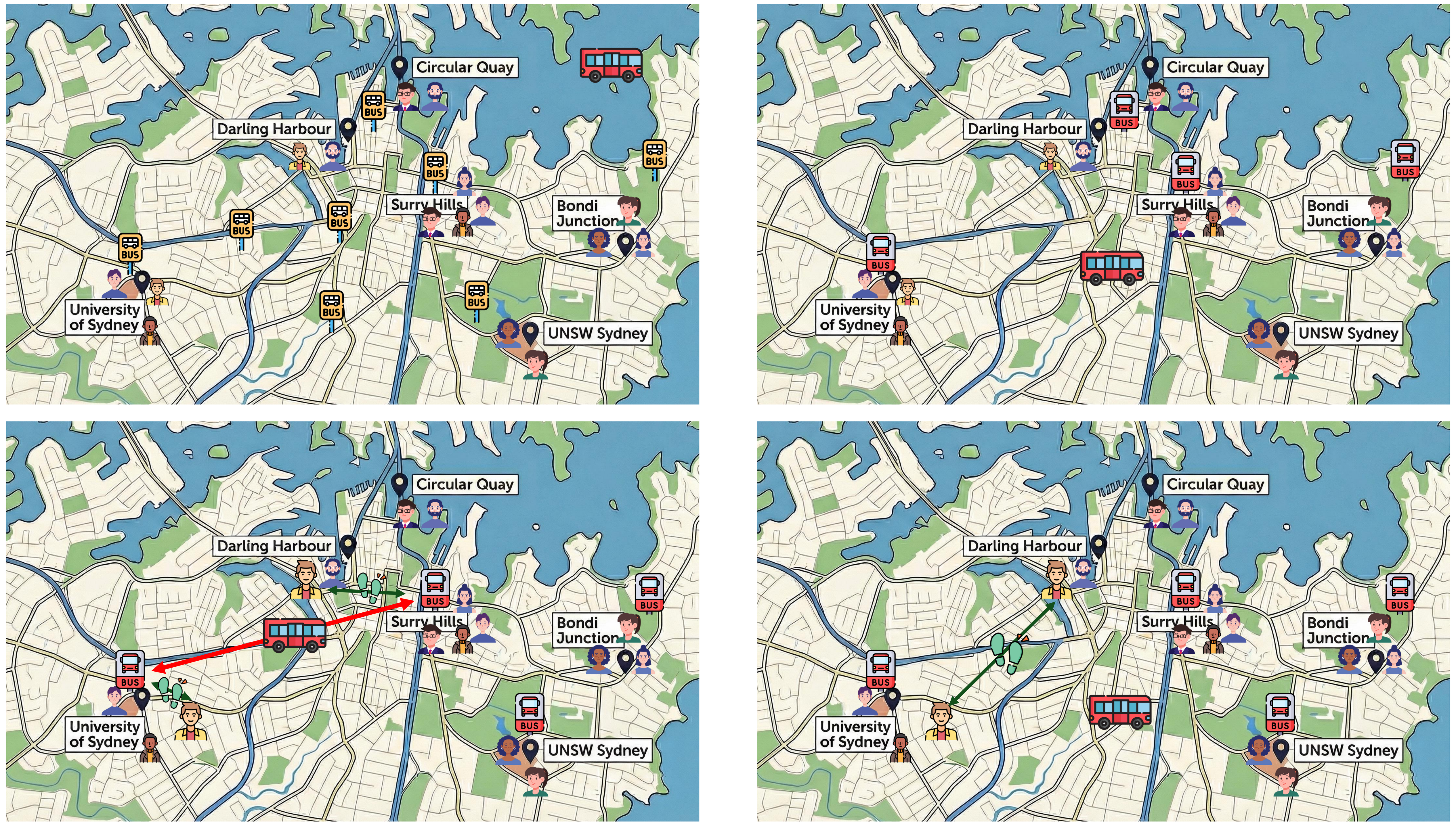
Introduction

We study the transit stop placement (TrSP) problem [1] in general metric spaces, where agents travel between **source–destination pairs** and may either walk directly or utilize a shuttle service via selected transit stops. We investigate fairness in TrSP through the lens of **justified representation** (JR) and the **core**, and uncover a structural correspondence with fair clustering.



Model

Consider a general metric space $(\mathcal{X}, d)$. Let $N := \{1, 2, \ldots, n\}$ denote a set of n agents and $\mathcal{C} \subseteq \mathcal{X}$ denote a set of m candidate stops. Each agent has a travel route $\theta_i = (a_i, b_i)$, where $a_i, b_i \in \mathcal{X}$.

- Input: A TrSP instance $I = \langle N, \mathcal{C}, \Theta = \{\theta_i\}_{i \in N}, k \rangle$.
- Output: A (placement) solution $Y \subseteq \mathcal{C}$, $|Y| \leq k$, where k is a positive integer.
- Define another metric $d' : \mathcal{C} \times \mathcal{C} \to \mathbb{R}_{\geq 0}$, representing the *transit system cost*.
- Given any feasible placement Y. The **cost** of agent i with $\theta_i = (a_i, b_i)$ is defined as

$$c_i(Y) = \min\left\{ \underbrace{d(a_i, b_i)}_{\text{walking cost}}, \underbrace{\min_{y_1, y_2 \in Y} \left[d(a_i, y_1) + d'(y_1, y_2) + d(y_2, b_i) \right]}_{\text{transit cost}} \right\}.$$

β-Justified Representation (β-JR)

Let $\beta \geq 1$. A placement $Y \subseteq \mathcal{C}$ is said to provide β-JR if there is no group of agents $S \subseteq N$ with $|S| \geq \frac{2n}{k}$, **a pair of transit stops** $T \subseteq \mathcal{C}, |T| = 2$, such that for each agent $i \in S$, $\beta \cdot c_i(T) < c_i(Y)$.

(α, β)-Core

Let $\alpha, \beta \geq 1$. A solution $Y \subseteq \mathcal{C}$ is in the (α, β)-core if there is no group of agents $S \subseteq N$, and **a transit stop set** $T \subseteq \mathcal{C}$ with $|S| \geq \alpha \cdot |T| \cdot \frac{n}{k}$, such that for each agent $i \in S$, $\beta \cdot c_i(T) < c_i(Y)$.

Fair TrSP Problem Meets Fair Clustering

ρ-Proportional Fairness [2] (ρ-PF)

Consider a general metric space $(\mathcal{X}, d)$, a set $\mathcal{N}$ of n datapoints, a set $\mathcal{M}$ of m candidate centers, and an integer $k > 0$. A clustering solution $Y \subseteq \mathcal{M}$ satisfies ρ-PF if there is no group $S \subseteq \mathcal{N}$ with $|S| \geq \frac{n}{k}$ such that there exists some $t \in \mathcal{M} \setminus Y$ such that for every $i \in S$, $\rho \cdot d(i, t) < \min_{y \in Y} d(i, y)$.

Reduction Result

- Given any ρ-PF clustering algorithm, there is an associated TrSP algorithm which achieves $(2, \rho)$-core.
- Given any β-JR TrSP algorithm, there is an associated clustering algorithm which achieves $(2\beta - \varepsilon)$-PF.

Algorithms and Approximation Results

Greedy Capture for TrSP (GC-TrSP)

1. Grow distance balls centered at each candidate stop; 2. Pick any stop whose ball contains at least $\lceil \frac{n}{k} \rceil$ active endpoints and deactivate them; 3. Terminate when all endpoints are deactivated.

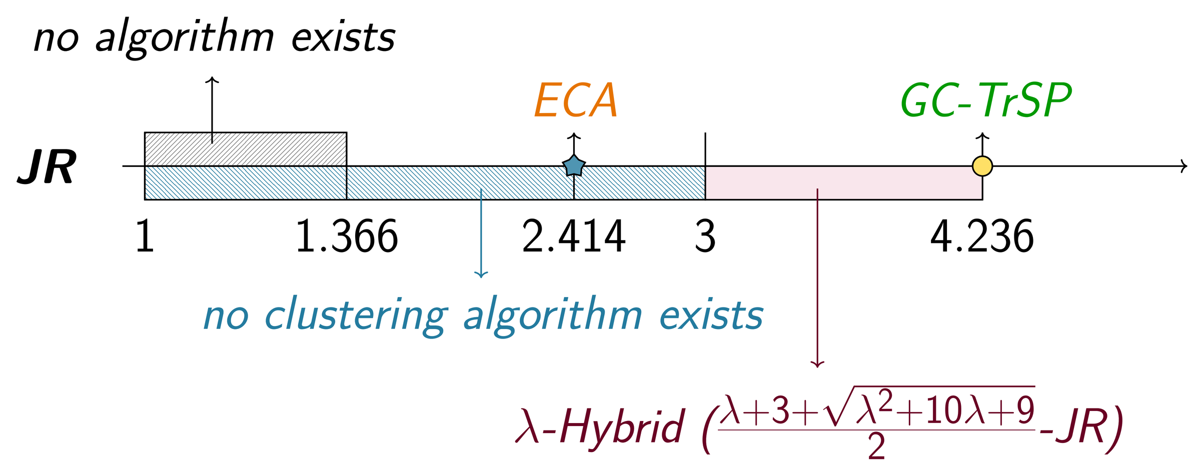
Expanding Cost Algorithm (ECA)

1. Grow cost balls centered at each pair of candidate stops; 2. Pick any pair whose cost ball covers at least $\lceil \frac{2n}{k} \rceil$ active agents and deactivate them; 3. Terminate when all agents are deactivated.

λ-Hybrid

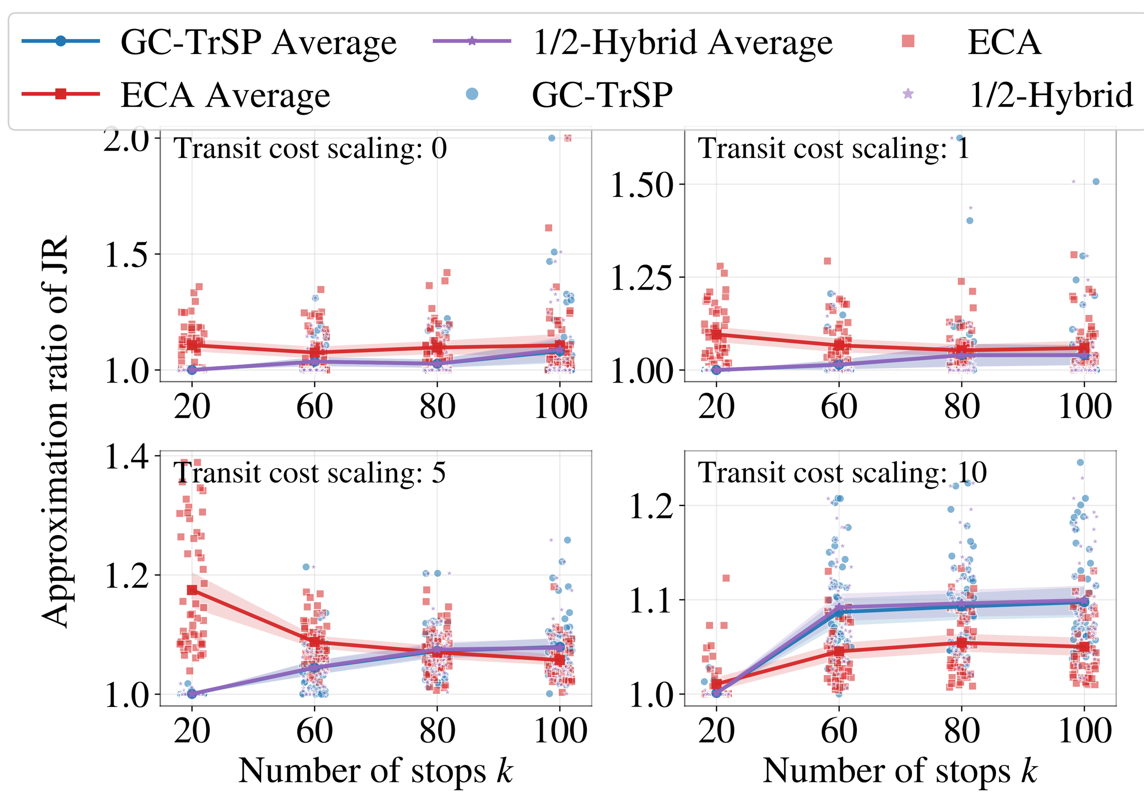
1. Grow distance balls $(\lambda \cdot r)$ at each candidate stop and cost balls (r) around each pair of stops simultaneously; 2. Pick any stop or pair whose ball contains at least $\lceil \frac{n}{k} \rceil$ endpoints or $\lceil \frac{2n}{k} \rceil$ agents and deactivate them.

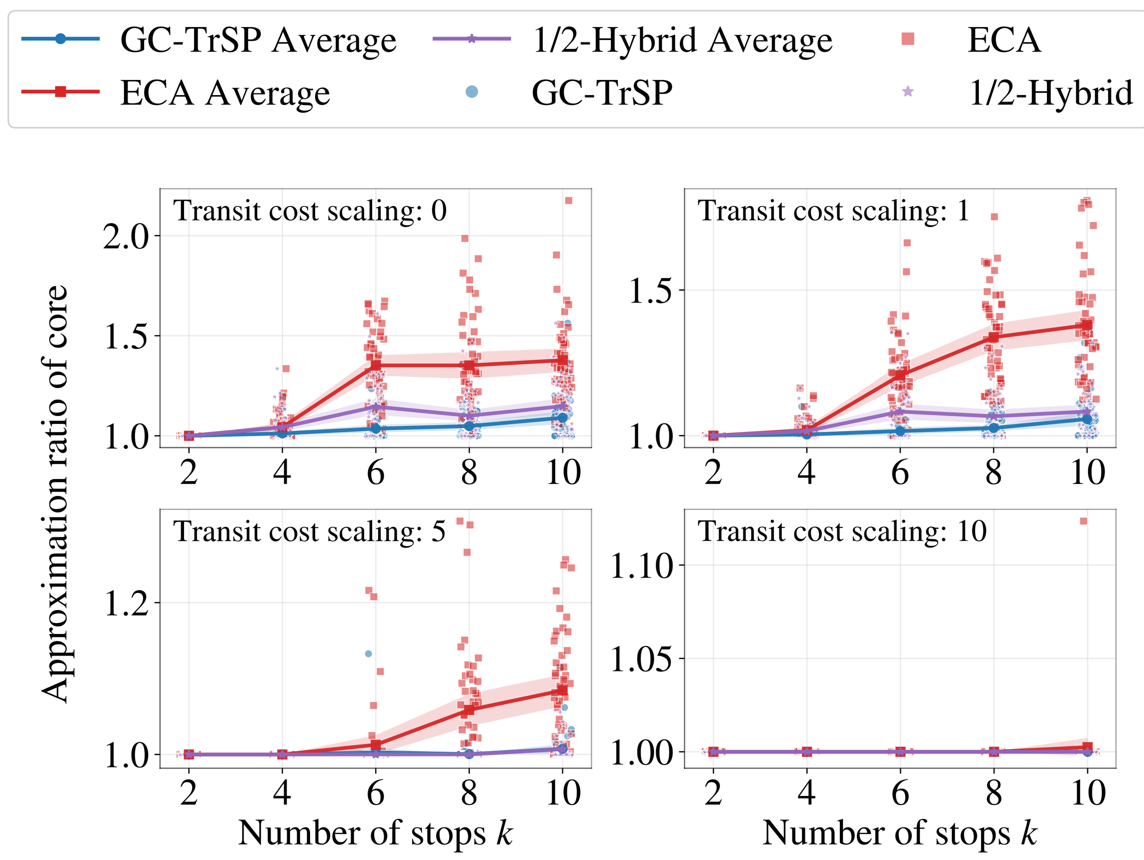
Theoretical Bounds



	core	
	Lower Bound	Upper Bound
GC-TrSP	$(2, 1+\sqrt{2})$	$(2, 1+\sqrt{2})$
ECA	(α, β)-core for any $\alpha, \beta \geq 1$	–
λ-Hybrid	$(2, \frac{\sqrt{4\lambda^2+12\lambda+1}+2\lambda+1}{4\lambda})$	$(2, \frac{\sqrt{\lambda^2+6\lambda+1}+\lambda+1}{2\lambda})$

Experimental Evaluation





References

[1] Martin Bullinger, Edith Elkind, and Mohamad Latifian. Towards fair and efficient public transportation: A bus stop model. In *Proceedings of the 24th International Conference on Autonomous Agents and Multiagent Systems (AAMAS)*, pages 427–435, 2025.

[2] Xingyu Chen, Brandon Fain, Liang Lyu, and Kamesh Munagala. Proportionally fair clustering. In *Proceedings of the 36th International Conference on Machine Learning (ICML)*, pages 1032–1041. PMLR, 2019.